A. Effect size and statistical power

Effect size (ES)
The effect size tells us something about how relevant the relationship between two variables is in practice. There are two types of effect sizes:
- Effect size based on the proportion of explained variance: the proportion of explained variance is often indicated by one of the following terms: R² or eta squared, partial eta squared or omega squared. These forms are discussed later in the summary.
- Effect size based on the difference in averages. This is often referred to using Cohen's d.

Calculating and using Cohen's d
Cohen's d is a common way to calculate the effect size and is calculated using one of the following formulas:

$$d = \frac{\bar{y} - \mu}{\sigma} \text{ or } d = \frac{\bar{y}1 - \bar{y}2}{\sigma}$$

This depends on whether you want to compare a sample mean with the population mean or two averages of two different samples. When we would choose the first method and thus want to compare the found sample mean with the population mean, the value of Cohen's d provides the number of standard deviations the sample mean is located from the population mean. So, when you would find a value of d = 0.5, the sample mean would be located 0.5 standard deviations from the population mean.

There is a number of rules of thumb that are usually used to determine whether an effect size is small, medium or large. An effect size around d = 0.80 is called a large effect. When the value of the effect size is approximately d = 0.50, it is seen as medium. We find an effect size of around d = 0.20 small. Keep in mind that these rules are not more than rules of thumb. There are no set rules and someone could also easily opt for other rules of thumb.

Calculating statistical power
When a test of significance is performed, there are four possible conclusions.

	H_0 is rejected	H_0 is not rejected
H_0 is true	Type I error (α)	1 – type I error
H_0 is not true	1 – type II error	Type II error (β)

Table 1: Type I and type II error

There is a **type I error** when H_0 is true, but it is rejected. So, a type I error is the unjustified rejection of the null hypothesis. This error is indicated by the sigma sign α. A **type II error** occurs when H_0 is not true, but it is not rejected. This means the null hypothesis is unjustly accepted as being correct. This error is indicated by the beta sign β. You are right when H_0 is true and you did not reject it. This can be calculated by means of the formula 1 – α. You are also right when H_0 is not true and you rejected it. This can be calculated by means of the formula 1 – β. This is the statistical power of the test.

This statistical power is the rightful rejection of the null hypothesis. You want the value of the statistical power to be as large as possible. When you, for example, find a significant outcome in your research and are rejecting the null hypotheses, you also want to do this rightfully.

To calculate the power, you first calculate the value of $\bar{x}$ under the wrong null hypothesis. You do this using the z-score. So you look at when you would reject this null hypothesis under the wrong null hypothesis. When you know the value of $\bar{x}$, you calculate the right probability of rejecting H_0, using the z-score by including the correct value of the average. Calculating the statistical power can be difficult to understand. It can therefore be best explained using an example.

Suppose you have a population that is divided with $X \sim N (\mu, 1)$.
In this population, you perform a significance test with an alpha of 0.05, whereby you test the following:
H_0: $\mu = 4.0$ $\qquad$ H_a: $\mu > 4.0$

There are 49 participants. However, we found out that μ is wrong: this is not 4.0 but 4.6. To calculate the power, we must first find out the value of $\bar{x}$ under the wrong null hypothesis to figure out when H_0 is rejected under the wrong null hypothesis. We know that in a one-sided test with $\alpha = 0,05$ the z-score is z = 1.645. Then:

$z = \frac{\bar{x}-\mu}{\alpha/\sqrt{n}} = \frac{\bar{x}-4,0}{1/7} = 1.645$
$\bar{x} - 4,0 = 0,235.$
$\bar{x} = 4,235.$

Now you can calculate the correct probability of rejecting H_0 ($H_a = 4.6$):

$P H_a (\bar{x} > 4,235) = P\,H_a = (\frac{\bar{x}-4,6}{1/7}) > \left(\frac{4,235-4,6}{1/7}\right)$
$P H_a (Z > -2,555) = 1 - 0.0054 = 0.9946.$

We can conclude that there is a probability of 99.46% that H_0 is correctly rejected when H_a: $\mu = 4.6$ is true.

The statistical power of a significance test depends on:
- The sample size (n): when n increases, the power increases;
- The significance level (α): when α increases, the power increases;
- The effect size (explained below): when the effect size increases, the power increases.

If the three values stated above are known, the power can be calculated using the software program G*Power. Another way to increase your power, is by decreasing the error variance. This can be done by making groups in your research more homogeneous or by adding an additional variable to your research.

There are two ways of analyzing the statistical power:
- A priori analysis: this method is preferred. The power is calculated before the research is carried out by entering the effect size, the significance level and the desired statistical pwer in the program G*Power. Then, the required sample size is calculated. So, the sample size is adapted to the power.
- Post hoc analysis: this method runs after the research is carried out. By entering the effect size, the significance level and the sample size, you can calculate the power of the research. This is suitable as a priori method for future research.

B. One-way ANOVA and its applications

T-test with two samples
The t-test is used when you want to compare two samples. The following hypotheses are tested:
H_0: μ1 = μ2
H_a: μ1 ≠ μ2 (this can also be one-sided instead of two-sided)

Hereby the population mean μ and standard deviation σ are unknown: they are approached with $\bar{y}$ and s. Remember the big difference between a t-test and z-test: for a t-test, the population standard deviation is unknown. In this course we generally assume equal standard deviations. This way we can use S_{pooled}: the pooled standard deviation. The test statistic for the t test is as follows:

$$t = \frac{\bar{y}1 - \bar{y}2}{sp\sqrt{\frac{1}{n1} + \frac{1}{n2}}}.$$

This test statistic has n1 + n2 - 2 degrees of freedom.

One-way ANOVA
Unfortunately, the use of the t-test is limited to comparing only two groups. In some cases we want to compare more than two groups. We then use ANOVA: ANalysis Of VAriance. When performing an ANOVA, we look at the differences between averages of I independent groups that are included in your research. The H_0 and H_a are as followed:

H_0: μ1 = μ2 = μ3 = μ4
H_a: at least one μ is different or not all μs are equal

Because the idea behind ANOVA is virtually equal to performing a t-test, in an ANOVA you compare, just as with conducting a t-test, the between-group variation with the within group variation. The between group variation is the variation between the groups you are comparing. This is the part that can be explained by the differences between groups. In the t-test formula above, this is what happens above the line in the fraction. After all, here you compare the differences between the two groups by substracting one average from the other average. The within group variation is the variation within the groups that you are comparing. This is the part of the unexplained variance that stays within the groups, called the error. In the formula above, this is what is stated below the line in the fraction. There is always variation in scores, because the scores of the subjects are never all the same. This is shown by the standard deviation. An ANOVA table made with SPSS looks like this:

Descriptives

hartslag

	N	Mean	Std. Deviation
control	14	91,3321	9,84239
experiment groep1	16	81,5526	10,28214
experiment groep2	15	70,8133	9,63519
Total	45	81,0154	12,79277

Table 1: Example of ANOVA Table SPSS: Descriptive Statistics.

ANOVA

hartslag

	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	3055,927	2	1527,963	15,483	,000
Within Groups	4144,894	42	98,688		
Total	7200,821	44			

Table 2: Example of ANOVA Table SPSS: ANOVA

The components of this ANOVA table from SPSS will be explained below.

The sum of squares
The variance for ANOVA is displayed in sum of squares. The sum of squares can be divided as follows: $SS_{total} = SS_{group} + SS_{error}$

SS_{total} is the total spread that you observe. The formula for SS_{total} is: $SST = \sum ij(yij - \bar{y})^2$. This can be calculated manually by multiplying the total variance of y by (n-1). It can also be calculated as follows:
$SS_{total} = SS_{group} + SS_{error}$
$SS_{total} = MS_{total} \times DF_{total}$

SS_{group} is the spread between groups. In other situations, this is also known as SS_{model} or $SS_{between}$. The formula for is: $SSG = \sum ij(\bar{y}i - \bar{y})^2$. This can be calculated manually by substracting the total average from the averages of the groups, squaring and then multiplying by n. You can than sum the results. It can also be calculated as follows:
$SS_{group} = SS_{total} - SS_{error}$
$SS_{goup} = MS_{group} \times DF_{group}$

SS_{error} is the spread within groups that cannot be explained. This is also called SS_{within}. The formula is: $SSE = \sum ij(yij - \bar{y}i)$. This can be calculated manually by multiplying the variance of each group by (n-1) of that group. If you've done this, you can sum the results. The sum of squares error can also be calculated as follows:
$SS_{error} = SS_{total} - SS_{group}$
$SS_{error} = MS_{error} \times DF_{error}$

If you have the sum of squares, you can use it together with the degrees of freedom (df) to calculate the mean square.

Degrees of freedom
The degrees of freedom can be calculated as follows:

$Df_{total} = n - 1$
$Df_{total} = df_{group} + df_{error}$
$Df_{total} = \frac{SST}{MST}$

Df_{group} (DFG) = I – 1, where I is the number of groups of X.
$Df_{group} = df_{total} - df_{error}$
$Df_{group} = \frac{SSG}{MSG}$.

Df$_{error}$ (DFE) = n − I.
Df$_{error}$ = df$_{total}$ - df$_{group}$
Df$_{error}$ = $\frac{SSE}{MSE}$